# bug detectives

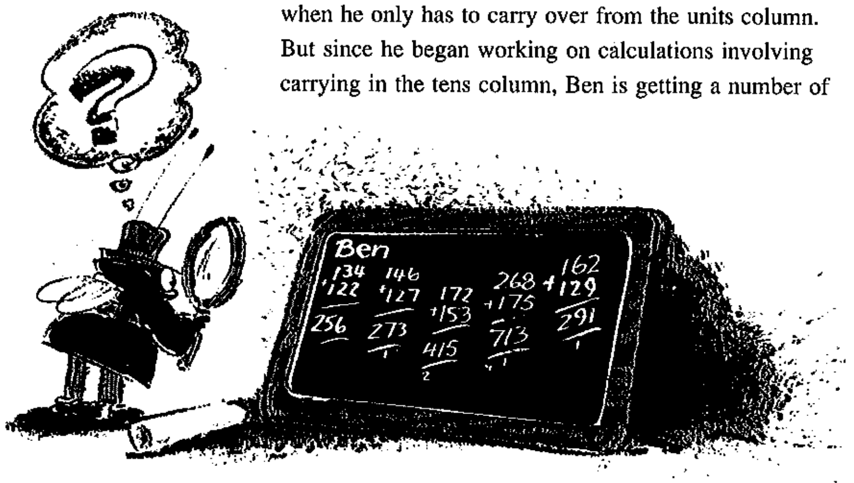
***Mike Askew explains how teachers can analyse their pupils' mathematics mistakes to correct systematic errors and prevent long-term failure***

*Although in* reading, miscue analysis is commonly used to gain insight into children's abilities, few teachers use children's calculation errors in mathematics as a source of information. Instead they see them as the result of carelessness, and believe that corrections are all that is needed. However, children are more likely to make mistakes because of a systematic error, or *bug*, in their thinking than out of carelessness.

### BEN

Ben makes typical 'bug' mistakes in his work. Many of his calculations are correct and he is secure in his knowledge of number bonds. He can add two three-digit numbers without carrying and has no problems when he only has to carry over from the units column. But since he began working on calculations involving carrying in the tens column, Ben is getting a number of them wrong. Look at his calculations in Figure 1. What do you think leads him to make the mistakes?

Figure 1




JAMES ALEXANDER

It is not difficult to see that Ben is confused about which digit to carry when he is working in the tens column but it is less clear what causes this confusion. This was only revealed when Ben was asked to explain his calculations. His explanation of his working out for 268 + 175 was:

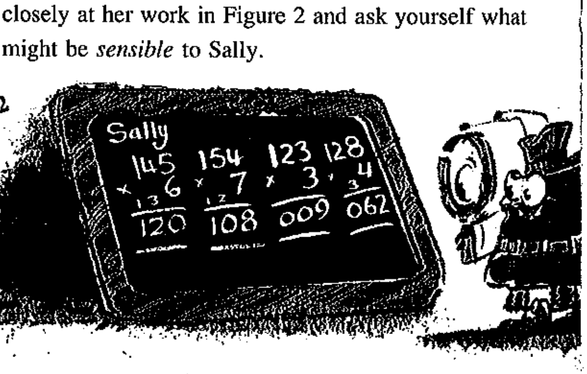
*Five add eight is thirteen. That's three units so put the three down and carry the one ten into the tens column. Six and seven and the one carried is fourteen. That's one ten and I'm in the tens column so put the one down and carry the four. Two and one and four is seven, so put that down.*

Ben is trying to remember the rules for doing such additions mechanically - but he has forgotten part of the instructions. In reconstructing the rules he uses some of his understanding of place value but he cannot see the error arising from talking about the digits in the tens column as though they were units. As a result, he has invented a procedure that, although mathematically incorrect, makes sense to him.

### SALLY

Sally's work demonstrates that children try to make sense of mathematical problems rather than simply doing things mechanically. Sally, a bright seven-year-old, has been given a practice workbook by her student teacher. The aim is to keep Sally occupied when she finishes the set work. Unlike Ben, Sally does not try to remember a procedure which she has previously been taught. When she comes across these calculations she invents a method for herself. At first glance, it may appear that she has very little understanding - but look closely at her work in Figure 2 and ask yourself what might be *sensible* to Sally.

Figure 2



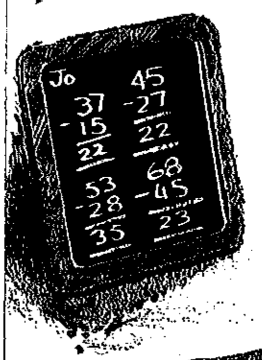
When the teacher talked to Sally about her method it became clear that she knew her tables and that she treated multiplication as: 'a bit like adding'. Looking back over her workbook the teacher could see that Sally could add three-digit numbers when they were set out vertically, like Ben's examples, and it seemed that she had adapted this method to deal with multiplication. So, to find the answer to 145 x 6 she would multiply the five by six, put down zero and carry the three. Because her experience of addition had involved doing things column by column, and she knows that multiplication is linked to addition, Sally then multiplied the four by three, putting down the two and carrying the one. Finally one times one is one. In those examples where nothing had been carried forward she simply multiplied by zero. While Sally's lack of awareness of the smallness of her answers shows a lack of understanding of multiplication, by inventing her own method she is acting more mathematically than if she simply had applied a procedure mechanically.

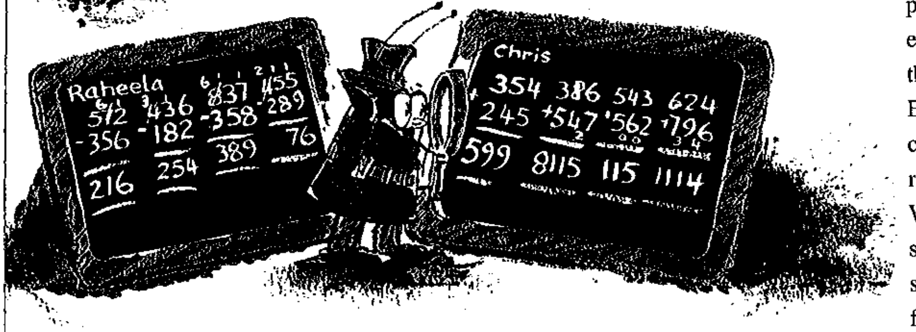
## *JO, RAHEELA AND CHRIS*

Now look at the examples in Figure 3 and ask yourself the following questions before discussing them with a colleague.

* Which method are the pupils applying?
* Is there a possible logic to the work from the pupils' point of view?
* What type of experiences might have sowed the seed for this bug?
* If I were the teacher what might I do next to help these children?
* What sort of strategies could I use with my class to see if any pupils have bugs in their understanding?

Figure 3

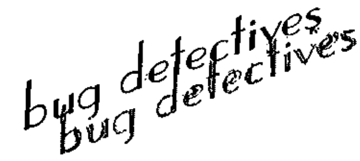



**Jo's bug in subtraction is probably the most common.**

**Until recently, Raheela has not displayed any difficulties with subtraction. She can use decomposition when it involves moving a ten into the units column. However, things begin to go wrong when she has to decompose in the hundreds column as well.**

**Chris has no difficulty in doing additions of hundreds, tens and units when no carrying is involved. After working out some examples involving carrying and using structured blocks, he tried to do some without the blocks.**




### Reinforcing the problem

*It* is not easy to change the views of children like those featured here, as the methods and explanations they create for themselves make sense. Simply marking solutions as right or wrong actually can reinforce the bug. Because not all the answers are wrong, marking provides a partial reward system and can reinforce the children's misunderstanding. Although going back over the right procedure could result in short-term improvement, unless the explanation is linked to the children's understanding of how they think numbers operate, the long-term results are unlikely to be successful. So, the starting point must be the children's explanations of how the mathematics operates. Once in the open, these can be examined and built upon. Asking children to explain out loud how they have worked things out can be very revealing but, this is often difficult for both teachers and pupils. Teachers tend to listen to the children's explanations to see how closely they match their own explanations rather than really to hear what they have to say. Meanwhile pupils can interpret a request to explain what they have done as a coded way of saying that something is wrong. And it is difficult to make time for one-to-one discussions when you have a class of 35. So sharing methods in a large group, or even as a whole class, is not only more practical, but may also achieve better results. It encourages the children to help each other and allows them to hear someone else describing a method. Emphasise the importance of estimating answers. If children realise that their method cannot possibly be right they have a powerful motivation to sort things out. Why not turn your pupils into bug detectives? Prepare a sneet of completed calculations that contain a systematic error - always taking the smaller number from the larger in subtraction for instance. Ask the children to mark them and explain to the fictional pupils what they are doing wrong. As we all know, you never really understand something until you try to teach it!

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